Architecture and mathematics in the time of Senusret I: Sections G, H and I of Papyrus Reisner I

Corinna Rossi
Collegio di Milano

Annette Imhausen
Johannes Gutenberg University Mainz

1 - Introduction

The fame of ancient Egypt in the minds of most people today is firmly built on the remains of its outstanding architecture, which bear witness to the remarkable abilities of ancient Egyptian architects. Egypt’s achievements in mathematics, while somewhat less famous, also generate intense interest. Egypt and Mesopotamia have the earliest surviving mathematical documents, providing insights into their mathematical culture.

However, the modern analysis of Egyptian achievements in both subjects has often been less than satisfying. Although the ruins of temples and pyramids should ‘speak for themselves’ – and to some degree do so, numerous questions arise to which the available material does not provide a straightforward answer. Lack of evidence and careless exploitation of scarce evidence have sometimes led to conclusions that will not stand the test of a careful analysis. However, exactly those false conclusions, which are often the most spectacular, have been the most persistent in modern minds.

An obvious example are the countless theories that claim to explain the geometry of the Great Pyramid at Giza on the basis of the most diverse and complex figures and proportions. These include triangles, squares, circles, heptagons, ratios between sides and height, sides and apothem, sides and corners, areas of faces and of the base, not to mention $\pi$ and especially $\varphi$, the Golden Section, always a favourite (for a summary, see Herz-Fischler 2000). There is still a strong resistance among the general public as well as in some scholarly circles (see, for instance, Romer 2007: 515 n.21) against the idea that all the Egyptian pyramids were built on the basis of a simple geometrical model: a right-angled triangle called seked (see below), derived from the Egyptian ‘sqd’ (to build), the Egyptian terminus technicus to describe sloped surfaces. The lack, until recently, of studies that would give mathematics its due place and role within ancient Egyptian culture has somehow given license to the apparent modern psychological need to search for “hidden secrets” in a distant past. Once we trust the scant but clear ancient sources, the interpretation of the geometry of the ancient monuments becomes immediately more straightforward and fruitful (Rossi 2004).

Likewise, the more than scarce source material of Egyptian mathematics has sometimes led to an equally skewed image, which over time has become accepted as truth by modern mathematicians and Egyptologists. Earliest historiography was based on the now outdated assumption that mathematics exists and develops independently of a social and cultural context. These studies focused mainly on similarities with modern mathematics, and used modern mathematical terminology and concepts in their analysis of the ancient texts. The result (not surprisingly) often showed how deficient Egyptian mathematics was (especially when compared to its mathematically mighty Mesopotamian neighbour). The very different source situation (about half a dozen mathematical texts from Egypt vs. thousands of

S. Ikram & A. Dodson (eds.), Beyond the Horizon: Studies in Egyptian Art, Archaeology and History in Honour of Barry J. Kemp (Cairo, 2009).
Mesopotamian mathematical texts) is only one reason why an approach of this kind has very little meaning indeed (for a sophisticated comparison of the mathematics of these two cultures see Ritter 1989). A new beginning has been made some time ago by Jim Ritter, indicating that the form and context of Egyptian and Mesopotamian mathematics need to be taken into account in order to obtain a better picture of them (Ritter 1989, 2000 and 2005). Although the number of sources for our knowledge of Egyptian mathematics has only marginally increased during the last several years (Imhausen and Ritter 2004 and Imhausen 2006a), there remain many aspects of it that have not as yet been explored (for an outline of the current situation, cf. Imhausen 2006b).

2 – Foundations of knowledge on architectural planning in ancient Egypt

In order to investigate the use of mathematics in ancient Egyptian architecture, a triple expertise is needed, which combines knowledge in Egyptology with that of the history of mathematics and the history of architecture. Even if the co-authoring team of this article is lucky enough to cover these three subjects, our investigation still faces a number of problems.

Only few ‘good’ sources exist. For the Old Kingdom, these are mainly indirect (i.e. the existence of ruins or a few isolated sketches), and the amount of information that can be deduced from such evidence is limited. It is certain that mathematical techniques were used in the construction of the pyramids at Giza, not only for the building process itself, but also to cope with the logistics of a project of this kind, i.e. the number of workers needed, their rations, etc. But it will remain unknown what exactly these mathematical techniques were, because the first detailed evidence in the form of mathematical papyri only dates from the Middle Kingdom.

The drawing on an Old Kingdom mastaba (see fig. 1) may indicate that the Egyptian concept of the \( \text{sqd} \) to describe slanted surfaces was in use by that time. However, the first concrete evidence mentioning the technical term and describing how it was calculated is found in the Rhind Mathematical Papyrus, which was written in the Second Intermediate Period.

From the Middle Kingdom onwards, mathematical texts exist that provide some insight into the kind of mathematics used in architecture. However, the extant mathematical papyri are surprisingly poor in architectural problems: there are only six (pBM10057–8 [\( \text{Rhind} \)], nos. 56–60 and pMoscow 4676, no. 14), all of which deal with some sort of pyramidal structure. In comparison, there are 21 problems related to baking and brewing, 15 problems concerning rations and 13 problems about calculations of areas. Evidence from the six architectural problems indicates that:

- the mathematical relation between base, height and inclination of a pyramid was known;
- the volume of a regular truncated pyramid could be calculated exactly;
- slanted surfaces were described by the \( \text{sqd} \), i.e. the horizontal displacement of the sloped face for a vertical drop of one cubit, that is, the distance by which the sloped side had ‘moved’ from the vertical at the height of one cubit (Rossi 2004: 185). The \( \text{sqd} \) was always indicated in palms and fingers.

The New Kingdom has left us practically no mathematical texts, but a good number of architectural sketches and written records of building activities survive, which provide important information on how mathematics was involved in the planning and building process (Rossi 2004: 104). It appears that buildings (either free-standing or rock-cut) were planned on the basis of simple figures (Glanville 1930; Hayes 1942: 15, pl. 7), but that changes (even substantial ones) during the construction process were perfectly acceptable, and were not given any particular importance. A typical example is the construction of...
rock-cut tombs: in the initial plan, rooms and corridors were given simple, round measures expressed in cubits (Engelbach 1927, Rossi 2001a; Rossi 2001b), but the final survey, meant to record that the work had been completed, recorded with extreme accuracy the natural ‘irregularity’ of the final dimensions in cubits, palms and even fingers (Carter and Gardiner 1917; Rossi 2004: 139–47).

In addition, the mathematical section of pBM10247 [pAnastasi I] (13,4–18,2) includes three problems referring to architecture: the construction of a ramp, the transport of an obelisk, and the installation of a colossal statue. For each of these, specific architectural details are given, e.g. the obelisk is said to be ‘of 110 cubits in the length of its shaft, its pedestal of 10 cubits, the circumference of its base makes 7 cubits on all its sides, its narrowing towards the summit 1 cubit, its pyramidion 1 cubit in height, its point 2 fingers’. However, none of them contains any indication how the respective problems were solved, and, indeed, the problems seem to be intentionally ‘underdetermined’, i.e. the numeric information given in the text is not enough to solve them. These examples were not intended to be actual mathematical problems for a junior scribe, like those of the mathematical texts; rather, they were meant to remind the readers of their former mathematical education.

The Reisner papyri, most notably Papyrus Reisner I, provide a remarkable addition to the corpus of sources. The set of four rolls of papyrus, now known as pReisner I–IV, were discovered by George Reisner during his first excavation of Naga el-Deir in 1904. They were left on top of one of three wooden coffins in tomb N408. After their discovery, they were unrolled in Berlin, then kept for some time at the Museum of Fine Arts in Boston, before they were given back to the University of California at Berkeley’s Center for the Tebtunis Papyri in the Bancroft Library in 2006. Why these papyri had been left in this tomb will probably remain an unsolved mystery (similar to that of the discarding of the Heqanakhte Papyri at Theban Tomb 315). Among the more obvious possibilities are that they were left accidentally while work on the tomb was in progress, or that they were left in this tomb intentionally as part of the possessions of one of the tomb owners (Simpson 1963: 17).

Papyrus Reisner I is approximately 3.5 m long and has a height of 31 cm. It is made up of nine sheets of papyrus, which are today mounted in seven frames (Simpson 1963: 18). Based on the regnal years indicated in the papyrus, the palaeography and the personal names, it has been dated by its editor, William Kelly Simpson, to the Middle Kingdom, more precisely to the reign of either Amenemhat I or Senusret I, with a preference for the latter (Simpson 1963: 19–21). In his edition, Simpson divided the document into various sections in order of their occurrence in the papyrus, which he designated by letters of the alphabet, beginning with the recto. He included black and white photographs, a hieroglyphic transcription, translation and a discussion of the content of the individual sections (Simpson 1963). Following their publication, the Reisner papyri have mainly been used to illustrate the administration of workers (see for example Simpson 1973, Mueller 1975 and Kadish 1996).

Imhausen had the opportunity to study pReisner I in 2002 [while it was still in Boston] during a fellowship at the Dibner Institute for the History of Science and Technology. For this she is indebted to the Museum of Fine Arts, and especially to Lawrence Berman for his assistance during her visit.
3 – Papyrus Reisner I and Egyptian mathematical practices (A. Imhausen)

Within pReisner I, sections G, H and I are of primary importance for our knowledge of mathematical practices during the Middle Kingdom. Each of these sections contains detailed records relating to the construction of a building. Although each section differs in the actual material which is being used, they contain the same type of numerical information, and are similar enough that lacunae in one section can be filled by information from the others.

3.1 – Tabular format of mathematical information

The first striking formal element to notice for the reader is a clear tabular format in which the data of some sections (G, H and I) are presented. Although there are so-called ‘table-texts’ within the mathematical papyri, most notably the $2\div N$ table, their layout is far less strict than one would expect.

For each division of $2\div N$, the table of the Rhind Mathematical Papyrus contains the ‘result’ consisting of a series of unit fractions, their respective size in terms of the divisor and a verification. The first two of these are written, alternating, for each term of the decomposition, and the verification is then added below each entry. The $2\div N$ table of the mathematical fragment UC32159 does not show the verification, but still lists the individual parts of the result and their respective sizes. This results in the $2\div N$ table resembling rather a list than a table. The same is true for the table of sums of fractions in the mathematical leather roll. In the leather roll, sums of fractions and their result are listed in four columns. There is no formal mathematical regularity that can be discerned throughout the arrangement.

However, in pReisner I a clear tabular format can be observed. Horizontal lines divide the whole of the text into several sections but, in addition, section A also shows vertical lines, thus creating an orderly grid system. The content of most sections is still that of a list; however, sections G, H and I are of ‘real’ tabular format. They are accounts detailing actual construction work, consisting of moving amounts of a material ($\text{hm}^\prime\text{w}$). The concrete meaning of $\text{hm}^\prime\text{w}$ is not known. Gardiner suggested ‘rubble’, which Simpson accepted, but added that it could also be ‘mud’ or ‘clay’ (Simpson 1963: 53).

Although formally only divided by horizontal lines, the seven columns of each section form a table in which the content of the last two columns is obtained via calculations from the first four columns. In addition, five of these six numeric columns also have headings above the first line of numerical information, which read $\text{sw}$ (length), $\text{wsh}$ (width), $\text{mdw}$ (depth), $\text{stti}$ (‘volume’ (?) $\text{iri m hsb}$ (identified as enlistees; Simpson 1963: 53). These headings and the accompanying numeric values render the interpretation of this section (including the fourth column, which does not have a heading) fairly straightforward as has been summarised by Simpson: ‘The fourth column has no heading, but it is clear that the figures in the column indicate the number of units or the number of times that the product of length, width, and thickness is to be multiplied to arrive at the volume. In brief, the columns of figures are to be read so that length (a), width (b), thickness (c), and number of units (d) are multiplied to determine the product (e), which is then divided by ten to arrive at the allowable number of man-days (f) for the task described in the entry’ (Simpson 1963: 53).

3.2 – The metrology of volume

The contents of sections G, H, and I further our knowledge of ancient Egyptian metrology in other ways as well. Volume calculations in the mathematical papyri refer almost
exclusively to amounts of grain (see pRhind, nos. 41–46 and UC 32160). In these problems, the metrological units used are those for grain products (for a discussion of these see now Pommerening 2005). Starting with linear dimensions of a granary indicated in cubits, the procedure to calculate the volume multiplies the respective measurements and then converts this intermediate into the respective capacity units, as for example in problem 44 of the Rhind Mathematical Papyrus: ‘Example of reckoning out a square container of 10 in its length, 10 in its breadth and 10 in its height. What is the amount that will go into it in corn? Multiply 10 by 10, it becomes 100. Multiply 100 by 10 it becomes 1000. Take a half of 1000, that is 500, it becomes 1500; this is its content in khar. You are to take a twentieth of 1500, it becomes 75. This is the amount that will go into it in quadruple-hekat, viz. 75 hundreds of quadruple-hekat of corn’ (Peet 1923: 84).

For easier reference and discussion, the procedure can be rewritten as follows (D = data; for the use of rewriting Egyptian mathematical procedures in this way see Ritter 1989 and Imhausen 2003):

\[
\begin{align*}
(1) & \quad 10 \times 10 = 100 \\
(2) & \quad 100 \times 10 = 1000 \\
(3) & \quad \frac{1}{2} \times 1000 = 500 \\
(4) & \quad 1000 + 500 = 1500 \\
(5) & \quad \frac{1}{20} \times 1500 = 75
\end{align*}
\]

The intermediate result (2) technically has the unit of a cubic-cubit; however, it is never explicitly stated as such. The first indication that the volume has been obtained is given in (4), after it has been converted into a capacity unit (khar), with the result given in another capacity measure (quadruple-hekat) at the end of the problem. Two of the granary problems (pRhind, no. 43 and UC 32160) even embed this conversion of units within the calculation to determine the volume (for a recent discussion of these problems see Imhausen 2003: 142–143 and 173 and Imhausen and Ritter 2004: 84–87).

The only volume calculation that is not related to grain is that of pMoscow, no. 14, the calculation of the volume of a truncated pyramid. However, nowhere in that problem is there any indication of the metrological units used. The length of the base is given as 4, the length of the upper side as 2 and the height as 6, resulting in a volume of 56.

Papyrus Reisner I provides the first explicit evidence for the use of the (cubic) cubit and its submultiples the (cubic) palm and (cubic) finger as metrological units. Not only are the results of the column headed stiti\(^{15}\) obtained by multiplying the previous columns, and we can hence assume the use of the cubic cubit (see table 1; stiti calculated was obtained by the author by a multiplication of the first four columns), but their subunits are explicitly stated in palms and fingers. For example, in section H, line 13, where a length of 2 (cubits) 3

\(^{13}\) Peet 1923 remains the best edition of the Rhind Mathematical Papyrus. For a discussion of the procedures of these problems see Imhausen 2003. A new edition of the Lahun mathematical fragments can be found in Imhausen and Ritter 2004 and Imhausen 2006.

\(^{14}\) The indication of cubits as given by Gillings (1972: 188) is not to be found in the source text (see Struve 1930: column XXVII and p. 135 with comment c) on p. 137: ‘In den geometrischen Aufgaben des M.P. wird nie der Name des Längenmasses angegeben’.

\(^{15}\) The meaning of ztti is not straightforward, and hence left untranslated here. For a preliminary discussion of the term, see Simpson (1963: 78–79).
ARCHITECTURE AND MATHEMATICS

palms, a width of 2 (cubits) 3 palms, and a depth of $\frac{2}{3}$ (cubit) results in a volume of 3 (cubic cubits), 6 (cubic) palms and $1\frac{1}{3}$ finger.

Throughout the three sections G, H and I, cubits themselves are never indicated explicitly – a number without a unit is assumed to be in cubits, whereas palms and fingers are explicitly indicated as such. It can be observed that there is no formal distinction between the linear units and the cubic units – cubic palms and fingers are written just like their linear counterparts. In a table like the one in pReisner I (and probably generally as well), this will not have caused any problems, as it is clear from the context (here the respective column) which of the two is meant.

For a remarkable amount of lines, the numerical values of all columns are extant and hence allow us to check the mathematical ability of the scribe of pReisner I. A number of entries use only cubits and fractions of the cubit (see table 1). In these cases, the majority of calculations are correct. However, if the indications of linear measurements also include the subunits palms and fingers, the numeric values that we find in the column indicating the volume present the reader with a problem that was first noticed by Richard Gillings (1972: 218–31). In those cases involving only cubits the result was mostly correct. In those cases that involved palms and fingers as well, the result was always wrong – if we assume that the Egyptian cubic units work like our modern units, i.e. that a cubic palm is a cube with the length, width and height of one palm.

In order to explain these deviations, Gillings suspected a sophisticated multiplication technique, devised by the author of pReisner I to satisfy his mathematically challenged superior for whom he prepared these accounts (Gillings 1972: 226). However, there is no indication whatsoever of the way that any of the calculations of pReisner I were executed. Nor would we expect these within an administrative text.

There may be a simpler method of explaining the numerical values found within the respective sections that does not require the above-mentioned assumptions, but primarily relies on the numerical values found in these tables. Let us take the example of section H, line 11 and compare our modern result of the volume calculation with the one we find in the source. The three measurements given in this line are 3 cubits 1 palm (length), 1 cubit (width) and 1 cubit (depth). The number of units is 1. If we were to calculate the volume from these data, we might transfer all these measurements into the smallest unit, in this case the palm, resulting in a length of 22 palms, a width of 7 palms, and a depth of 7 palms. The multiplication of these three values will result in the respective volume (in modern-concept cubic-palms), i.e. 1078. To transfer this back into cubic cubits it has to be divided by 343, resulting in $\approx 3.14$, that is, 3 cubits 49 palms.

The final result given in the last column of pReisner I, however, is 3 cubits 1 palm and it is quite obvious that the ancient calculation must have been different from our modern interpretation. If we assume the result of H11 as correct, the deviation from our modern expectations can be explained by a different concept of subunits for volume-cubits. Instead of conceiving the smaller units as cubes with a side that equals the respective smaller linear measure, they are conceived as rectangular prisms. That is, the volume palm

---

10 Some of Gillings’ conclusions are identical to the results of the explanation that is suggested in this article. However, since the former are too deeply embedded in Gillings’ system of reconstructing arithmetical techniques for which we have no evidence at all, it is difficult to disentangle the areas where we come to the same conclusions.

17 In addition to the linear measurements the number 1/3 has been written after the indication of the depth. Additional notes occur occasionally within these tables: they may refer to some calculation associated with the data of the table, but do not have an influence on the calculation of other entries of the table itself.
is a rectangular prism with the dimensions 1 cubit, 1 cubit and 1 palm (see fig. 2). This way of conceptualising units of measurements is in line with the Egyptian metrology of areas as well, as is demonstrated by the existence of the area-cubit (or 'cubit-of-land'), which is a strip of 100 cubits by one cubit.¹⁸

If we re-examine the lines involving the subunits with this concept in mind, it becomes clear that a number of results found in sections H and I are correct (5 out of 14), and some (6 out of 14) are ‘almost correct’ (see table 2). Further observations can be made: in all of the correct results, only one of the linear measurements involved a subunit, the other two were given in cubits only.

As stated initially, we do not have any evidence indicating how the individual calculations were performed. Gillings suggested a multiplication method using what he calls ‘scales of notation’. Another possibility might be that all of the linear measurements were converted into parts of the cubit, possibly using auxiliary numbers to ease the technically difficult handling of fractions – which of course had to be reconverted at some point. At some stage of these calculations, rounding might have been used to simplify an individual calculation, hence explaining the slight deviations from the exact result. Mostly, these deviations are less than 1 (Egyptian) cubic palm. The fact that all of the cases in which only one linear measurement involved a subunit are correct (lines H11, H17, I8, I9 and I10) points to the fact that these calculations were easier to perform than those involving more than one subunit.

This way of handling smaller units is not only very sensible for calculations, because it obviously avoids cumbersome calculations with large numbers (i.e. multiplications and divisions by 343 (cubit to palm), 64 (palm to finger) or even 21,952 (cubit to finger); it is also useful for practical considerations, because it is easier to imagine these standard prisms with a square base of one cubit by one cubit and different heights than to handle cubes of these small sizes.

4 Papyrus Reisner I and Egyptian architecture (C. Rossi)

The building described in pReisner I remains, at least for the moment, impossible to identify, as too many variables of the equation are unknown. First of all, the terminology employed might refer to a temple, a cenotaph or a tomb (Simpson 1963: 22). Then the meaning of the crucial word ḫmāw is unclear (Simpson 1963: 53). Moreover, architecture was not the primary concern of the scribe: his aim was to calculate how many man-days were necessary to complete a list of tasks, that is, the focus was organisation of the workforce, rather than recording the building process (compare, for instance, the opposite case of ostracon Strasbourg H.112; Koenig 1997: 9). As a consequence, it is possible that pReisner lists only parts of a building, thus making the identification even more difficult.

All these doubts combine with a basic problem: the lack of substantial architectural remains of Middle Kingdom temples. A wealth of inscriptions, statues and loose or re-used blocks indicate that the kings of this period were prolific builders (Petrie 1896; Mond and Myers 1940; Habachi 1972). However, apart from a few exceptions (Spencer 1989: 63) in the subsequent historical periods, Middle Kingdom temples were either used as quarries (a good example is the Labyrinth: Petrie, Wainwright and Mackay 1912; Michalowski 1968; Lloyd 1970; Arnold 1979), or swept off by later expansions and new buildings (one example, among many others, may be Karnak: Gabolde 1998; Blyth 2006: 10–26).

Senusret I, the king under whom pReisner is likely to have been compiled, is credited with the construction of temples all over Egypt, from Heliopolis (Daressy 1903; Petrie and

¹⁸ The area cubit is attested in problem 55 of the Rhind mathematical papyrus, see Peet 1923: 25 and 96–97).
Mackay 1915; Ricke 1935) to the Fayum (Chaaban 1926), from his major funerary complex at Lisht (Arnold 1988 and 1992) to the area of the First Cataract (Habachi 1975; Schenkel 1975). He also built a substantial temple at Abydos, a few kilometres south of the site where pReisner was found: several inscribed and decorated blocks suggest that this building must have been impressive, but all that remains of its layout is little more than a rectangular outline and a few foundation deposits (Petrie 1903).

In general, because of this lack of basic information, reconstructing the appearance of Middle Kingdom temples can be rather difficult, especially when the building was made of mudbricks combined with a few stone elements (compare, for instance, Bisson de la Roque 1937: fig. 6 with Arnold 1975: fig. 4). An increasingly higher percentage of stone facilitates the task, and the apex of this process is represented by the complete and successful reconstruction of the White Chapel of Senusret I (Lacau and Chevrier 1956; see also Naumann 1939 and Wegner 1995).

Because of all of these problems, the architectural interpretation of pReisner I appears to be rather difficult – but not impossible. Of course many important questions must remain open, first of all the identification of the building described, but nevertheless several interesting details may be deduced from the available data.

4.1 – Architectural interpretation of Section G
Section G contains a list of volumes of ḫmāw to be dealt with by a certain number of workmen. Since the meaning of ḫmāw is unclear, it is also unclear whether it had to be removed or added to the building. The same term also occurs in an inscription from the Satet temple of Elephantine built by the same king under which pReisner was compiled, Senusret I. In that case, the ḫmāw is said to be 20 cubits (Schenkel 1975: 124).

A partial solution to this riddle may be suggested by the actual shape of the volumes: apart from the one described in line 2 (which appears to be 7 cubits deep), and those of lines 3 and 4 (the third dimension is missing), all the others appear to be relatively ‘thin’ geometrical figures, with a depth ranging from 1/4 of a cubit to 2 cubits (see fig. 3). If we visualise these volumes on the ground, the most logical interpretation is that they are shallow trenches of varying depth, meant to accommodate the foundations of various parts of the building. The deepest trench might have accommodated foundations of particularly heavy elements, such as large stone columns.

A good example of this technique can be found in the temple which Senusret I built at Tôd: a picture published by Bisson de la Roque clearly shows the appearance of the mud platform into which shallow areas of various shape were cut to accommodate walls and pillar bases (Bisson de la Roque 1937: fig 5). Figure 4 contains a schematic version of this foundation platform, with the original dimensions of some of these trenches given by Bisson de la Roque in metres and their ‘translation’ into cubits.

It is interesting to note that the volumes listed in Section G are invariably expressed using fractions of cubit, rather than palms and fingers. Therefore, the dimensions of the trenches of the temple at Tôd, if possible, have been also expressed in fractions of cubits. It may be worth noting that the photograph seems to suggest that the depth of the trenches might be more irregular than the neat -10 cm and -20 cm given by Bisson de la Roque. In fact, this is also true for the other measurements, and for this reason the ‘translation’ into cubits, palms and fingers of the dimensions in metres should be allowed a certain flexibility.

The two sets of dimensions, those of pReisner and those of Tôd, although referring to different buildings, show a remarkable similarity. Section G may therefore contain a list of volumes to be removed in order to accommodate walls and other architectural elements of
a building. There is also the possibility that ḫw directly or indirectly referred to the sand layer that was used to stabilise the foundation of buildings. There is evidence that this old method was used also in the Middle Kingdom (for example at Tôd: Bisson de la Roque 1937: 5 onwards), but the dimensions and characteristics of the sand layer vary considerably from building to building (Arnold 1991: 110 onwards). Even if Reisner refers to a specifically codified method to fill with sand the separate foundations of the various elements of a building (cf. Arnold 1991: 114–5), the basic concept remains the same, as in this case the volume to be filled must have been dug or modelled first.

4.2 – Architectural interpretation of Sections H and I

Section H contains a list of stone blocks to be transported by a certain number of workmen. When the dimensions are preserved, the blocks appear to be all rather elongated, with a length ranging between 1 1/2 cubit (ca 79 cm) and 4 cubits + 4 palms (2.40 m), a width ranging between 6 palms (45 cm) and 1 cubit + 1 palm (60 cm) and a depth ranging from 1/2 cubit (ca 26 cm) to 1 cubit + 2 palms (ca 67 cm). It is interesting that the dimensions of stone blocks are expressed either using fractions of cubits (as in Section G) or using palms and fingers. Lines 30 to 35 contain again volumes of sand similar to those listed in Section G; in particular, the elements described in lines 33 and 34 are identical (fig. 3).

Rather than ‘normal’ blocks to be used to build a wall, the stones described in Section H might have been used in specific contexts: for instance, the blocks described in lines 17, 23 and 28 (the largest of the list) and perhaps also those of lines 6 and 4 (incomplete), which are 4 cubits to 4 cubits + 4 palms long (2.10 to 2.40 m), may be architraves. A parallel may be offered by the temple at Tôd, where the doorways of the inner part appear to have been 1.35 m wide, that is, about 2 1/2 cubits (Bisson de la Roque 1937: 9); in this case, adding a bit less than 1 cubit on each side to allow the architrave to rest on the doorjambs, the total length of the architrave might have been about 4 cubits. The other blocks listed in Section H might have been used for other architectural details, such as doorjambs and lintels for smaller doorways.

This interpretation agrees with the scant information that we possess on the structure of several early Middle Kingdom temples, which appear to have been mainly built with a combination of mudbrick walls and stone elements (Arnold 1975: 186). Examples of this type of construction are the temples at Tôd built by Mentuhotep II and III (Arnold 1975: figs. 1 and 3); the temple at Ezbet Roushti built by Amenemhat I and enlarged by Senusret III (Bietak, Dorner, Czerny, Bagh 1998) and the temple at Hermopolis (Roeder 1937: 12–7, Plan 2), beside the already mentioned temple at Tôd built by Senusret I.

Section I mentions, in fact, the use of bricks in lines 12 and 18 to 21. The operations described in this section appear to be related to various stages of brickwork, but it is difficult to provide any additional comment. In general, it may be noted that lines 2 to 8 list long, narrow and thin volumes (length between 8 and 52 cubits, breadth between 3 and 11 cubits, thickness between 1/4 and 2/3 of a cubit); lines 9 to 12 describe long, narrow but thicker volumes (length between 20 and 26 cubits, breadth between 5 and 7 cubits, thickness between 5 palms and 2 cubits); line 8 describes smaller volume (8 × 7 × 2 cubits), whereas lines 14 to 17 list small thick volumes (length between 1 1/4 and 4 cubits, breadth of either 1 1/2 or 2 1/2 cubits and thickness of either 1 1/2 or 2 cubits); finally, lines 18 to 20 describe slightly larger thick volumes (length between 8 and 16, breadth between 5 1/2 and 6 cubits and thickness between 1 of a cubit and 6 palms). It is likely that at least some of these volumes referred to mud-brick walls, but it is impossible to say whether they belonged to the building itself or to supporting or additional structures.
5 – Conclusions

The analysis of the mathematics of sections G, H and I of pReisner I adds to our understanding of Egyptian mathematics. The use of a clear tabular format to render mathematical data and results of operations based on them is especially interesting since the mathematical table par excellence, the $2 \div N$ table, does not show this format in either of the two preserved copies (pRhind and UCL 32159).

The conceptualisation of subunits in volume metrology is yet another indication of the practical focus of mathematics in ancient Egypt. Breaking down the cubic-cubit into subunits measuring 1 cubit $\times$ 1 cubit $\times$ 1 palm, and 1 cubit $\times$ 1 cubit $\times$ 1 finger may have enabled an easier assessment of the actual amount of material that was being handled, rather than using cubic-palms and cubic-fingers: ‘slices’ of cubits would have been easier to handle than any other type of sub-unit. One would assume that such a definition of the subunits also resulted in a somewhat technically less demanding arithmetic in the process of calculating volumes, but how exactly the calculations were carried out remains unknown. An explanation would ideally use information taken from the mathematical papyri on how multiplications (and divisions) were carried out, and be able to explain the deviations or rounded results found in section H.

The observations gained from the mathematical analysis can be used in the architectural interpretation, and further our understanding of this text. For instance, the dimensions of Section G are expressed by whole numbers of cubits with or without the addition of basic fractions ($\frac{1}{2}$, $\frac{1}{4}$ and $\frac{2}{3}$): this appears to confirm the impression that the operation described here is digging rough trenches meant to accommodate the foundations of a number of architectural elements. A higher level of accuracy was instead required when dealing with such elements, as the lines of Section H listing stone blocks clearly indicate: here the dimensions are expressed in cubits, palms and fingers, and even fractions of the latter. Indirectly, this accuracy suggests the existence of a detailed architectural plan, according to which the quarrymen produced blocks of specific dimensions.

In conclusion, pReisner might refer to a temple made of mudbrick walls with the addition of stone elements such as lintels and doorjambs, resting on a foundation platform into which shallow trenches were prepared to accommodate the superstructure. A precise identification with an existing building is currently impossible, but other contemporary temples provide interesting parallels. The planning and building process of this temple was carried out on the basis of accurate arithmetical calculations, meant to establish and keep under control the size of the various parts of the building and also to organise the workforce in the most productive way. This document clearly attests the deep involvement of mathematics at all levels of a construction process, and confirms that understanding the ancient mathematical mechanisms may help understanding the fields in which mathematics was applied, and vice versa. For us, it confirms the importance of interdisciplinary research.

BIBLIOGRAPHY


HAYES, W. C. 1942. Ostraka and Name Stones from the Tomb of Sen-mut (No. 71) at Thebes (New York: MMA).


LACAU, P. and CHEVRIER, H. 1956. Une chapelle de Sèsostris Ier à Karnak (Cairo: IFAO).


NAUMANN, R. 1939. ‘Der Tempel des Mittleren Reiches in Medinet Madi’, MDAIK 8: 185–89.


STRUVE, V. 1930. Mathematischer Papyrus des Staatlichen Museums der Schönen Künste in Moskau (Berlin: Springer).
<table>
<thead>
<tr>
<th>Line</th>
<th>3w</th>
<th>wsh</th>
<th>mdw</th>
<th>units</th>
<th>stti (text)</th>
<th>stti (calc.)</th>
<th>✓/×</th>
</tr>
</thead>
<tbody>
<tr>
<td>section G</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>5</td>
<td>$\frac{1}{4}$</td>
<td>1</td>
<td>10</td>
<td>10</td>
<td>✓</td>
</tr>
<tr>
<td>11</td>
<td>32</td>
<td>4</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>85</td>
<td>64</td>
<td>×</td>
</tr>
<tr>
<td>15</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>48</td>
<td>48</td>
<td>✓</td>
</tr>
<tr>
<td>section H</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$\frac{21}{3}$</td>
<td>$\frac{11}{3}$</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>1$\frac{1}{2}$</td>
<td>1$\frac{1}{4}$</td>
<td>✓</td>
</tr>
<tr>
<td>25</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>1</td>
<td>2</td>
<td>2$\frac{1}{3}$</td>
<td>✓</td>
</tr>
<tr>
<td>26</td>
<td>$\frac{21}{3}$</td>
<td>$\frac{11}{3}$</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>2</td>
<td>3$\frac{1}{4}$</td>
<td>✓</td>
</tr>
<tr>
<td>27</td>
<td>$\frac{31}{3}$</td>
<td>$\frac{11}{3}$</td>
<td>$\frac{1}{2}$</td>
<td>2</td>
<td>15$\frac{1}{4}$</td>
<td>15$\frac{1}{4}$</td>
<td>✓</td>
</tr>
<tr>
<td>30</td>
<td>12</td>
<td>5</td>
<td>$\frac{1}{4}$</td>
<td>1</td>
<td>15</td>
<td>15</td>
<td>✓</td>
</tr>
<tr>
<td>31</td>
<td>15</td>
<td>5</td>
<td>$\frac{1}{4}$</td>
<td>1</td>
<td>18$\frac{1}{4}$</td>
<td>18$\frac{1}{4}$</td>
<td>✓</td>
</tr>
<tr>
<td>32</td>
<td>15</td>
<td>5</td>
<td>$\frac{1}{4}$</td>
<td>1</td>
<td>18$\frac{1}{4}$</td>
<td>18$\frac{1}{4}$</td>
<td>✓</td>
</tr>
<tr>
<td>33</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>64</td>
<td>64</td>
<td>✓</td>
</tr>
<tr>
<td>34</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>36</td>
<td>36</td>
<td>✓</td>
</tr>
<tr>
<td>35</td>
<td>8</td>
<td>5</td>
<td>$\frac{1}{4}$</td>
<td>1</td>
<td>18</td>
<td>10</td>
<td>×</td>
</tr>
<tr>
<td>section I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>5</td>
<td>$\frac{1}{3}$</td>
<td>1</td>
<td>30</td>
<td>30</td>
<td>✓</td>
</tr>
<tr>
<td>12</td>
<td>27</td>
<td>7</td>
<td>2</td>
<td>1</td>
<td>378</td>
<td>378</td>
<td>✓</td>
</tr>
<tr>
<td>13</td>
<td>8</td>
<td>7</td>
<td>2</td>
<td>1</td>
<td>112</td>
<td>112</td>
<td>✓</td>
</tr>
<tr>
<td>14</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>$\frac{1}{3}$</td>
<td>2</td>
<td>9</td>
<td>9</td>
<td>✓</td>
</tr>
<tr>
<td>15</td>
<td>$\frac{21}{3}$</td>
<td>$\frac{11}{3}$</td>
<td>$\frac{1}{2}$</td>
<td>2</td>
<td>11$\frac{1}{4}$</td>
<td>11$\frac{1}{4}$</td>
<td>✓</td>
</tr>
<tr>
<td>16</td>
<td>$\frac{31}{3}$</td>
<td>$\frac{11}{3}$</td>
<td>$\frac{1}{2}$</td>
<td>2</td>
<td>25$\frac{1}{3}$</td>
<td>26$\frac{1}{3}$</td>
<td>×</td>
</tr>
<tr>
<td>17</td>
<td>4</td>
<td>2</td>
<td>$\frac{1}{2}$</td>
<td>2</td>
<td>36</td>
<td>30</td>
<td>×</td>
</tr>
</tbody>
</table>

**TABLE 1**

Numeric values of complete lines in sections G, H, and I using cubits only
(✓ correct result; × mistake).
TABLE 2

Numeric values of complete lines in H, and I using cubits, palms and fingers (note that section G uses cubits only): ✓ correct result; (✓) small deviation (< 1 palm); × mistake. In the column ‘stti (calculated)’ Egyptian fractions were used where easily possible, otherwise the result is indicated using our fractions.

<table>
<thead>
<tr>
<th>line</th>
<th>δw</th>
<th>ws</th>
<th>md</th>
<th>units</th>
<th>stti (text)</th>
<th>stti (calc.)</th>
<th>comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>section H</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>3 1p</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3 1p</td>
<td>3 1p</td>
<td>✓</td>
</tr>
<tr>
<td>13</td>
<td>2 3p</td>
<td>2 3p</td>
<td>(\frac{2}{3})</td>
<td>1</td>
<td>3 6p 1 (\frac{1}{3}) f</td>
<td>3 6p</td>
<td>(✓)</td>
</tr>
<tr>
<td>14</td>
<td>2 2f</td>
<td>1 (\frac{1}{2})</td>
<td>1 1p 1f</td>
<td>1</td>
<td>3 4p 1 (\frac{1}{3}) f</td>
<td>3 4p 2 (\frac{15}{28}) f</td>
<td>(✓)</td>
</tr>
<tr>
<td>15</td>
<td>1 5p</td>
<td>1 (\frac{1}{2})</td>
<td>5p</td>
<td>2</td>
<td>3 5p</td>
<td>3 4p 2 (\frac{5}{7}) f</td>
<td>(✓)</td>
</tr>
<tr>
<td>16</td>
<td>2 3p</td>
<td>1 4p</td>
<td>5p 2f</td>
<td>1</td>
<td>2 5p 2 (\frac{1}{2}) f</td>
<td>2 6p 3</td>
<td>×</td>
</tr>
<tr>
<td>17</td>
<td>4</td>
<td>1 3p</td>
<td>1</td>
<td>4</td>
<td>22 6p</td>
<td>22 6p</td>
<td>✓</td>
</tr>
<tr>
<td>18</td>
<td>3 2p</td>
<td>1 2p</td>
<td>6p</td>
<td>1</td>
<td>3 3p 2 (\frac{1}{3}) f</td>
<td>3 4p 1 (\frac{19}{39}) f</td>
<td>(✓)</td>
</tr>
<tr>
<td>19</td>
<td>3 5p 2f</td>
<td>1 3p</td>
<td>1</td>
<td>1</td>
<td>4 2p 3f</td>
<td>5 2p 3 (\frac{1}{3}) f</td>
<td>×</td>
</tr>
<tr>
<td>20</td>
<td>3 3p</td>
<td>1 3p</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>4 6 (\frac{1}{4})</td>
<td>(\frac{1}{3}) p</td>
</tr>
<tr>
<td>23</td>
<td>4</td>
<td>1 6p</td>
<td>6p</td>
<td>1</td>
<td>4 1f</td>
<td>6 2p</td>
<td>×</td>
</tr>
<tr>
<td>24</td>
<td>3 5p</td>
<td>1 2p</td>
<td>6p</td>
<td>1</td>
<td>4 2f</td>
<td>4 2 (\frac{30}{39}) f</td>
<td>(✓)</td>
</tr>
<tr>
<td>section I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>24</td>
<td>5p</td>
<td>(\frac{1}{2})</td>
<td>1</td>
<td>8 4p</td>
<td>8 4p</td>
<td>✓</td>
</tr>
<tr>
<td>9</td>
<td>26</td>
<td>6</td>
<td>5p</td>
<td>1</td>
<td>111 3p</td>
<td>11 3p</td>
<td>✓</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>5</td>
<td>5p</td>
<td>1</td>
<td>71 3p</td>
<td>71 3p</td>
<td>✓</td>
</tr>
</tbody>
</table>
FIG. 1
Sketch of the ancient guide-lines drawn on subterranean retaining walls at the four corners of Mastaba 17 at Meidum (4th Dynasty).
From Petrie 1892: pl. viii, courtesy Petrie Museum of Egyptian Archaeology, UCL.

FIG. 2.
Modern cubic units vs. Egyptian volume units: ancient cubic cubit, modern idea of cubic palm and Egyptian volume palm ($c =$ cubit; $p =$ palm).
Volumes listed in Sections G and H; depth in brackets, visualised with varying shades of grey; unit of measurement: cubit.

Fig. 3

Temple of Senusret I at Tôd

Schematic plan of the foundation platform of the temple at Tôd, with dimensions in metres and cubits; depth visualised by shades of grey.